MARKSCHEME

MAY 2016

MYP MATHEMATICS EXTENDED

ON-SCREEN EXAMINATION

24 pages

The markscheme may make use of the following abbreviations:

RM Assessor has the following annotations that should be used to award marks:

A0 only use to award a zero mark for an answer that has no merit e.g. awarded for the candidate that has a wrong answer with no working

NR only use when the candidate has not made any response also stamp the response with **SEEN**

✔1 Marks awarded by stamping the tick

SEEN Seen; must be stamped on all blank response areas and on concatenated responses

❓ unclear

- Bullet notation means award 1 mark – see example 1 below

ECF Marks that can be awarded as **error carried forward** from previous results in the question
BOD Benefit of the doubt
MR misread
NWS no working shown
SC special case
OE or equivalent
WTTE or words to that effect
AG Answer given

Example 1
- 1 mark awarded and corresponding notes are aligned

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b	• Show clear line of reasoning in the method	45 & 49 seen **OE** e.g. $49 = 45 + x$	2
	• 4	**ACCEPT** 45+X/10=4.9 **and** Ans 4	

Error Carried Forward (ECF) Marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.
   a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
   b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or sinx > 1) then subsequent marks should not be awarded.
   c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
   d) To award **ECF** marks for a question part, **there must be working present for that part**.
   e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
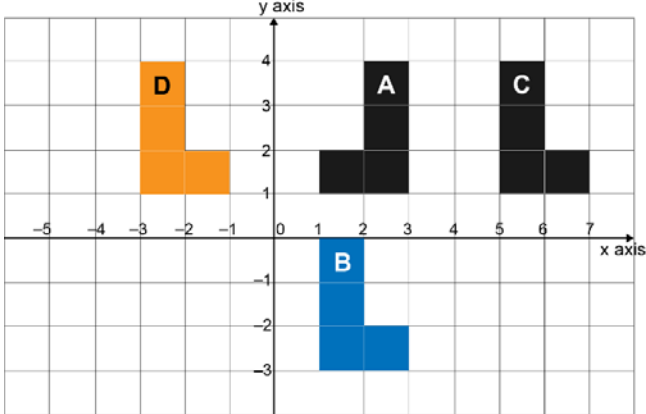   f) A misread (**MR**) is an error**. ECF** is normally awarded.

General points

   a) As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
   b) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,** then the last mark cannot be awarded.
   c) Where candidates have written two solutions to a question, mark the response that deserves more marks.
   d) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** or equivalent e.g. $\dfrac{1}{2}$ or 1/2 or $1 \div 2$ and $\dfrac{x}{2}$ $x/2$ or $x \div 2$

   e) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
   f) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.

Questions marked using Holistic markscheme in task 3 questions.

Some questions in the markscheme are indicated to be marked holistically. In these questions follow the following procedure to award the marks:
   1) Best-fit the work in a certain band by taking an overview of the first two columns, the strand and the specified criteria.
   2) Use best judgement to award the work the specific mark within the band; whether in lower, middle (if appropriate), or highest.

Task 1

		Answers	Notes	Total
1	a	• $x = 4$		**1**
	b	• $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$	Accept (4,4) **or** p = 4 and q = 4	**1**
	c		Must be correct within tolerance of half a square Accept no label on D	**1**
	d	•¹ $4 \times -4 + 4 \times 4$ **or** vector format $\begin{pmatrix} 4 \\ 4 \end{pmatrix}\begin{pmatrix} -4 \\ 4 \end{pmatrix}$ •² 0	•¹ **Their** answer (b) and $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ which is given **OE** •² **ECF** for correct multiplication of **any** two vectors	**2**
	e	• explanation that mentions 90º or perpendicular	**ECF** could be awarded for a correct comment on **their** product e.g. correctly state the angle is acute/obtuse. For example the candidate says : "It allows you to find the angle between the vectors" "it is the cosine of the angle between the vectors" "the angle is X i.e. they correctly find **their** angle"	**1**

2	a	•¹ 1st mark for two from the points below •² 2nd mark for four from the points below Correct comment comparing median or LQ or UQ or min or max Correct comment comparing IQR or range Correct values of median or LQ or UQ or min or max Correct values of IQR or range	A comment for comparison would have higher or lower or same in both countries **or WTTE** Any reference to mode e.g. "most/usually shoe sizes" is incorrect Accept use of average for the median Accept values of IQR given as (LQ-UQ) Accept values of range given from min to max **SC** If there is not enough comments or correct values but the candidate states: that the shoes sizes in Brazil are bigger than in Egypt **WTTE**: award 1 mark	2
	b	•¹ one multiplication correct •² second multiplication and addition shown •³ correct answer 0.08	•¹ **OE** 6 % or 0.06 seen or 2 % or 0.02 •² **OE** 0.06+0.02 •³ **OE** 0.08 without working award 2 marks	3
	c	•¹ $P(B/D) = \dfrac{P(B \cap D)}{P(D)}$ •² $\dfrac{0.4 \times 0.05}{\text{their } 0.08}$ •³ $= \dfrac{1}{4}$ **OE e.g. 0.25**	•¹ Use of conditional probability formula •² Accept their 0.02/**their** 0.08 **ECF** **Note:** Formula at •¹ can be implied All probabilities must be less than 1 •³ $\dfrac{1}{4}$ **OE** without working: award 2 marks	3

3	a	$\bullet^1 \; f \circ g(x) = \dfrac{x+3-2}{3(x+3)-11}$	$\bullet^1$ Substitutes x + 3	2
		$\bullet^2 \quad = \dfrac{x+3-2}{3x+9-11}$	$\bullet^2$ Opens out bracket	
		$= \dfrac{(x+1)}{(3x-2)} \quad$ **AG**	**Note:** if they use $g \circ f$: award 0 marks	
	b	$\bullet^1 \;\; 3yx - 2y = x + 1$	$\bullet^1$ Changes $f \circ g$ *fog* to y **and** cross multiplies	6
		$\bullet^2 \;\; 3yx - x = 2y + 1$	$\bullet^2$ x terms to same side	
		$\bullet^3 \;\; x(3y - 1) = 2y + 1$	$\bullet^3$ Taking x (or y if candidate exchanged them) as common factor	
		$\bullet^4 \;\; x = \dfrac{2y+1}{3y-1}$	$\bullet^4$ Writing x in terms of y	
		$\bullet^5 \;\; (f \circ g)^{-1}(x) = \dfrac{2x+1}{3x-1} \; ,$	$\bullet^5$ Exchanging x and y may be seen earlier	
		$\bullet^6 \;\; x \neq \dfrac{1}{3}$	$\bullet^6$ The domain can be written differently as $x \in \mathbf{R} \neq \{\dfrac{1}{3}\}$ or R - {1/3}	
			Note: Some working may be implied. **Note:** If an error makes the working easier full **ECF** cannot be awarded $(f \circ g)^{-1}(x) = \dfrac{2x+1}{3x-1} \; ,$ with no working : award 4 of first 5 marks	

3	b	Alternative method		
		$\bullet^1 \ y = \dfrac{x+1}{3x-2}$	$\bullet^1$ Changes $(f \circ g)(x) \ to \ y$	
		$\bullet^2 \ \dfrac{x+1}{3x-2}$	$\bullet^2$ Divides of the terms	
		$\bullet^3 \ h = \dfrac{2}{3}$ and $k = \dfrac{1}{3}$ or $y = \dfrac{5/9}{x-2/3} + \dfrac{1}{3}$	$\bullet^3$ Recognises h and k	
		$\bullet^4$ new $h = \dfrac{1}{3}$	$\bullet^4$ Puts h instead of k	
		$\bullet^5$ placing $1/3$ for h or $y = \dfrac{5/9}{x-1/3} + \dfrac{2}{3}$	$\bullet^5$ Puts k instead of h	
		$\bullet^6$ Dom : $x \neq \dfrac{1}{3}$	$\bullet^6$ Domain can be written differently	

4			Accept the use of decimals in their working. The final answer must be exact. See note at mark •⁹	
		•¹ $r^2 = 2^2 + 2^2$ or sin45 =2/r **OE**	•¹ Correct use of Pythagoras or trigonometry	
		•² $(r=)\sqrt{8}$	•² √8 SEEN Without working award 1 mark from •¹ and •²	
		•³ height of cone 4	•³ finds height of cone (may be seen at any stage or implied)	
		•⁴ $(Vcylinder =)\pi \times$ their $r^2 \times 26$	•⁴ Substitutes in formula	
		•⁵ $(v =)208\pi$	•⁵ 208π SEEN Without working award 1 mark from •⁴ and •⁵	
		•⁶ $(Vcone =)\dfrac{1}{3}\pi \times$ their $r^2 \times 4$	•⁶ Substitutes in formula	9
		•⁷ $(Vcone =)\dfrac{32}{3}\pi$	•⁷ 32 π /3 SEEN Without working award 1 mark from •⁶ and •⁷	
		•⁸ **their** cylinder + **their** cone	•⁸ Adding **their** two volumes. May be implied.	
		•⁹ $\dfrac{656\pi}{3}$	•⁹ Answer must be given in terms of π. Do not award •⁹ for an answer that is not exact.	
			SC: for the candidates that use r = 2 correctly: award 7 marks See below answers for this **SC** $(Vcylinder =)208\pi$ $(Vcone =)\dfrac{32}{3}\pi$ $(Total\ V =)\dfrac{656\pi}{3}$	

5	a	$\bullet^1$ crosses at 0 and 30 or a is a distance travelled along the x-axis $\bullet_2$ $x=0$ or $x=a$	$\bullet^1$ or when the ball is at 30 the height is 0 **OE** e.g. substitutes (30, 0)	2
	b	$\bullet^1$ the maximum height occurs at $x=15$ $\bullet^2$ $h(15)=9\,\mathrm{m}$	$\bullet^1$ Can be seen as substitution in the equation $\bullet^2$ 9 with no working award 1 mark	2
	c	$\bullet^1$ substitute $x=26$ $\bullet^2$ $h=4.16$ $\bullet^3$ 4.16 > 3 m therefore **SC**ore or yes	 $\bullet^2$ 4.16 without working: award 1 mark $\bullet^3$ 4.16 > 3 m, without working award 2 marks	3
	d	$\bullet_1$ $\dfrac{x(30-x)}{25}$ $\bullet^2$ $x(30-x)>75$ or $x^2-30x-75<0$ this should be $+75$ $\bullet^3$ $x=\dfrac{30\pm\sqrt{(-30)^2-4(1)(75)}}{2}$ $\bullet^4$ $(x=)27.2$ and $(x=)2.8$ $\bullet_5$ $27.2-2.8=24.4$	$\bullet^1$ sets up inequation $\bullet^2$ cross multiplies is minimum for the mark $\bullet^3$ substitutes into quadratic formula $\bullet^4$ finds 2 roots **or** estimates from the graph in the range 2<x<4 and 26<x<28 award: 1 mark from the first four bullets $\bullet^5$ subtracts to find the distance **or ECF** correctly subtracts **their** estimated values	5

Task 2

		Answers	Notes	Total
6	a	• the values double	**or** $r = 2$	**1**
	b	•¹ evaluates 3520 •² because the values are above 20 and below 20 000	3520 must be seen Accept "because 20<**their** A<20 000" **OE** Accept **their** 3520 is less than 20 000 Award 2 marks	**2**
	c	•¹ 1 term correct •² both terms correct and multiplied $(F =) 27.5^2 \times 2^n$	•¹ $F_n = F_0 \times 2^n$ award 1 mark •² Allow alternative fully correct formula i.e. $55 \times 2^{n-1}$	**2**
	d	•¹ $\dfrac{F}{27.5} = 2^n$ •² $\log_2 \dfrac{F}{27.5} = \log_2 2^n$ •³ $n = \log_2 \dfrac{F}{27.5}$	•¹ Rearrange •² Award 1 mark for $\log \dfrac{F_n}{27.5}$ •³ Award 2 marks for $\dfrac{\log \dfrac{F_n}{27.5}}{\log_2 2}$ **OE** •³ Correct answer without working: award 2 marks	**3**
	e	•¹ $n = \log_2 \dfrac{28160}{27.5}$ •² $n = \log_2 1024 = 10$ yes as a whole value	Alternative method •¹ Continue the sequence •² 10 seen and answer yes	**2**

	f	This table gives some key values which might be seen – the table goes over two pages					

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Aspect	1 mark	2 marks
IR: Identification of relevant information	One numerical factor mentioned from: – Octave (either width 194.5mm or number of octaves 9) – Keys (either width 23mm **or** number 63-66) – Arm span 1.65m – Width of the piano as shown 148cm – Room/door size (eg. average room 3x4) – Human hearing 20HZ to 20 000HZ	More than one numerical factor mentioned
CM: Calculation of measurements including width	Relevant calculation without mentioning gaps **or** extra width Examples: Calculating the number of keys ie 52 + 2 x7 Accept answer is in the range 63-66 **OR** Calculating the width of the piano keys (63-66)x 23 ACCEPT any reasonable value	Calculating width including gap **or** adding extra width Examples: 9 x 194.5 9 octaves and includes gap between keys gap width: (194.5 – 8x23)/ 7 **or** 8 gaps = 1.5 / ----mm Key + its gap = 23 + 1.5=24.5 Hence width = 66x24.5 + 1.5 (1 extra gap at the end) =1618.5 mm **OR** Calculating extra width at the end in the original piano: In normal piano of width 1480mm there are 52 keys so extra width both sides is 1480 – (52x24.5 + 1.5)=204.5 mm So the 66 keys piano needs to be 1618.5 mm + 204.5 = 1823mm
JD: Justification of degree of accuracy	Rounding used in any element	Justifies **their** choice of rounding
PD: Practicality of new design	Limited argument: Some examples: Not acceptable because people are used to smaller pianos Acceptable as it will include all octaves that people can hear	Justified argument Some examples: Referring to price in anyway: Not acceptable because the added octaves will increase the price of the machine and at the same time 2 octaves will not add much Referring to the difference in width between normal piano and this one in any way: Acceptable because the added width will be only about 30 cm which can fit in a standard room width
QD: Quality of overall discussion (Holistic judgement on the whole response)	Limited discussion	Balanced discussion

10

7	a	$\bullet^1$ $\cos(73) = \dfrac{1.19}{s}$ $\bullet^2$ $s = \dfrac{1.19}{\cos(73)} (= 4.07 \text{ m})$ **AG**		**2**
	b	$\bullet^1$ $\tan 73 = \dfrac{h}{1.19}$ $\bullet^2$ $(h =) 3.89 \ldots (2314616 \text{ cm})$	**OE** with sine ratio $\bullet^2$ Correct answer without working award 1 mark only	**2**
	c	$\bullet^1$ calculates angle 20 $\bullet^2$ $\dfrac{x}{\sin 20} = \dfrac{4.07}{\sin 53}$ $\bullet^3$ $(x =) 1.74 \ldots (3000362 \text{ cm})$ $\bullet^4$ $w_2 = 55.3 - 2 \times (\text{their } 1.74 + 1.19)$ **or their** 2.93 from $\bullet^3$ $\bullet^5$ $w_2 = 49.44 (\text{cm})$	 $\bullet^2$ Use of sine rule $\bullet^3$ Calculate x Accept correct method using other lengths Alternative method for $\bullet^1$ $\bullet^2$ $\bullet^3$ $\bullet^1$ TAN = h/y uses TOH in right angle triangle $\bullet^2$ y = tan/h cross multiply y = tan 53/3.89 **their** 3.89 $\bullet^3$ y = 2.93 calculate	**5**
	d	$\bullet^1$ attempt to work out another width **ECF** answer from (c) $\bullet^2$ $w_9 = 55.3 - 2(8) \times (\text{their } 1.74 + 1.19)$ $\bullet^3$ $w_9 = 8.42 (\text{cm})$	$\bullet^1$ $w_3 = 49.44 - 2 \times (\text{their } 1.74 + 1.19)$ $\bullet^2$ 8 subtractions $\bullet^3$ Accept correct to 1 decimal place	**3**

For candidates that do not apply the scale 1:100 treat as a misread **MR** on the first occasion, **ECF** can be awarded so candidates can gain full marks for subsequent working.
The final answer for 7a) 7b) 7c) should be in the answer box. If there is not a value in the answer box, award marks for final answers in the response box.

7	e	Aspect	1 mark	2 marks	
		Identify the relevant information required	Identify one of the following: Total measurements of the model Volume of the model Elements of the pyramid (temple and platform) the scale 1:100 Practicality of the size of the model	Identify more than one of the following: Total measurements of the model Volume of the model Elements of the pyramid (temple and platform) the scale 1:100 Practicality of the size of the model	
		Consider the degree of accuracy	Consideration the implication of the degree of accuracy on the given values. Examples: Recognizing that the lengths given to the nearest cm Recognizing that the angles given to one decimal place	Consideration the implication of the degree of accuracy on the calculated values. Examples: 2.7x9+6 =30.3 and not 30cm (2.5 instead of 2.7 we get 2.5x9+6=28.5 which is far from 30 **or** 3cm instead of 2.7 we get 3x9+6=33 which is far from 30) The width of first platform 55.3cm while it could be 55 and in this case 9th platform 55-2x2x8=23 cm The width of 9th platform could be 23 cm instead of 23.3 and in this case the 1st platform 55cm Recognizing the difficulty of using a model with dimensions more accurate than nearest mm because of measuring tools available	**6**
		Comment on the validity	Comment not supported with evidence. Examples: The model will be close enough to look like the real pyramid The model was calculated with correct mathematical steps so it is valid	Comment supported with evidences. Examples: The model is practical and referring to overall size (height being about 30, width being about 50 cm) and can be carried/placed on a table easily The model is not very valid because the total height needs to be to the nearest mm like the height of the platform (27 mm). this will lead to inaccurate angles compared to real pyramid The erosion had an effect on the real pyramid and this is not taken into account in the model	

Task 3

		Answers	Notes	Total
8	a	• states the limit as 8	Correct limit	**1**
	b	$\bullet^1$ two correct from p=5 q=6 r=7 $\bullet^2$ all three correct	Award 1 mark for two correct values	**2**
	c	$\bullet^1$ any correct patterns $\bullet^2$ in words give a rule e.g. divide column a by 3 and then add 2	For example Column a increases by 3 **or** multiples of 3 Column L increase by 1 Do not accept column b are the same / all 3	**2**
	d	$\bullet^1$ $L = \dfrac{1}{3}a + b$ or $L = xc + 2$ $\bullet^2$ $L = \dfrac{1}{3}a + 2$	Any correct rearrangement Award 1 mark for each correct term	**2**

	e	Mark holistically				

Strand	Holistic markscheme	Mark band
Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Nothing from below	**0**
Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	One prediction made Attempt to describe a pattern in words	**1 - 3**
Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern correctly described in words Attempt to test **their** described pattern	**4 - 6**
Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern described as suggested general rule consistent with some of the findings **Their** general rule is tested correctly	**7 - 9**
Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	More than one prediction made A pattern described as correct general rule consistent with findings A general rule is fully proved *or* verified and justified	**10 - 12**

12

SC More than one prediction made with constant value for column A without any pattern description award 3 marks

Patterns need to refer to a relation involving a b and L

Predictions could be seen in the table **or** in the answer box

Exemplification

The candidates can populate the table using the simulator.

Candidates who restrict themselves to the pattern shown can only discover simple patterns e.g.

For **L** in terms of row number [**L** = n+3] **and** **a** in terms of row number [**a** = 3(n+1)]. Simple patterns can only gain credit up to Mark band 7–9.

Candidates who use the simulator to vary **a** and **b** will be able to described more complex patterns and find $\frac{1}{3}a + \frac{2}{3}b$.

Using random values of **a** and **b** will probably not prove useful. Some systematic approach will be more helpful. Here are some examples.

Candidates keep the same **a** values and change **b** – they will soon see that multiples of 3 are better to use.

	a	*b*	*L*
	3	6	5
	6	6	6
	9	6	7
	12	6	8

They might write down a rule and test it again using the simulator.

They might try **b = 0**

	a	*b*	*L*
	3	0	1
	6	0	2
	9	0	3
	12	0	4

Candidates need to use other **a** values. They might keep **a** constant and vary **b**. And again put **a = 0**

	a	*b*	*L*
	3	3	3
	3	6	5
	3	9	7
	3	12	9

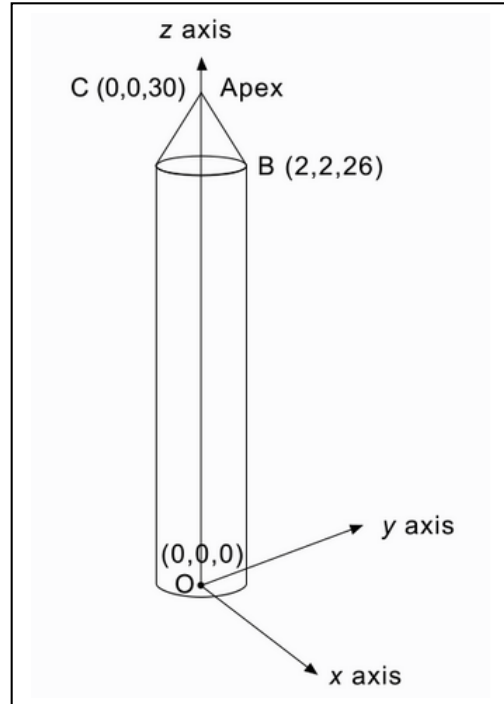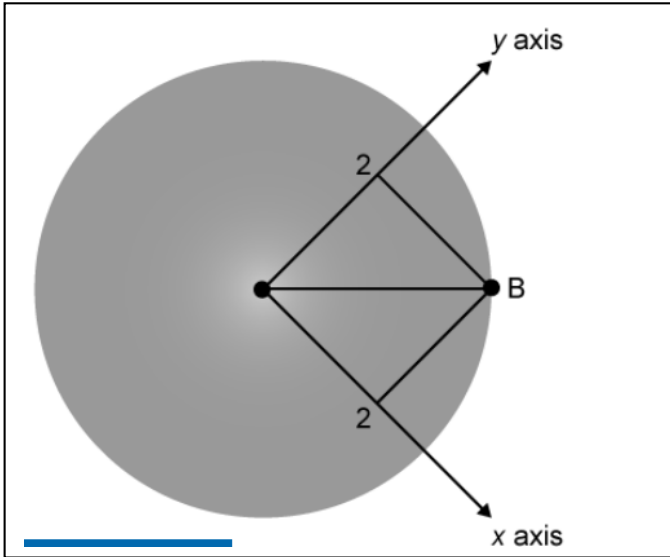|	*a*	*b*	*L*
	0	3	2
	0	6	4
	0	9	6
	0	12	8

9	a	$\bullet^1$ $\left(b + \dfrac{1}{2}a + \dfrac{1}{2}b\right) \div 2$ $\bullet^2$ $\left(\dfrac{1}{2}a + 1\dfrac{1}{2}b\right) \div 2 = \dfrac{1}{4}a + \dfrac{3}{4}b$ **AG**		**2**
	b	$\bullet^1$ student generates a sequence $\bullet^2$ $\dfrac{3}{4}$ $\bullet^3$ $\dfrac{5}{8}$ $\bullet^4$ this sequence represents the coefficients of **b**		**4**

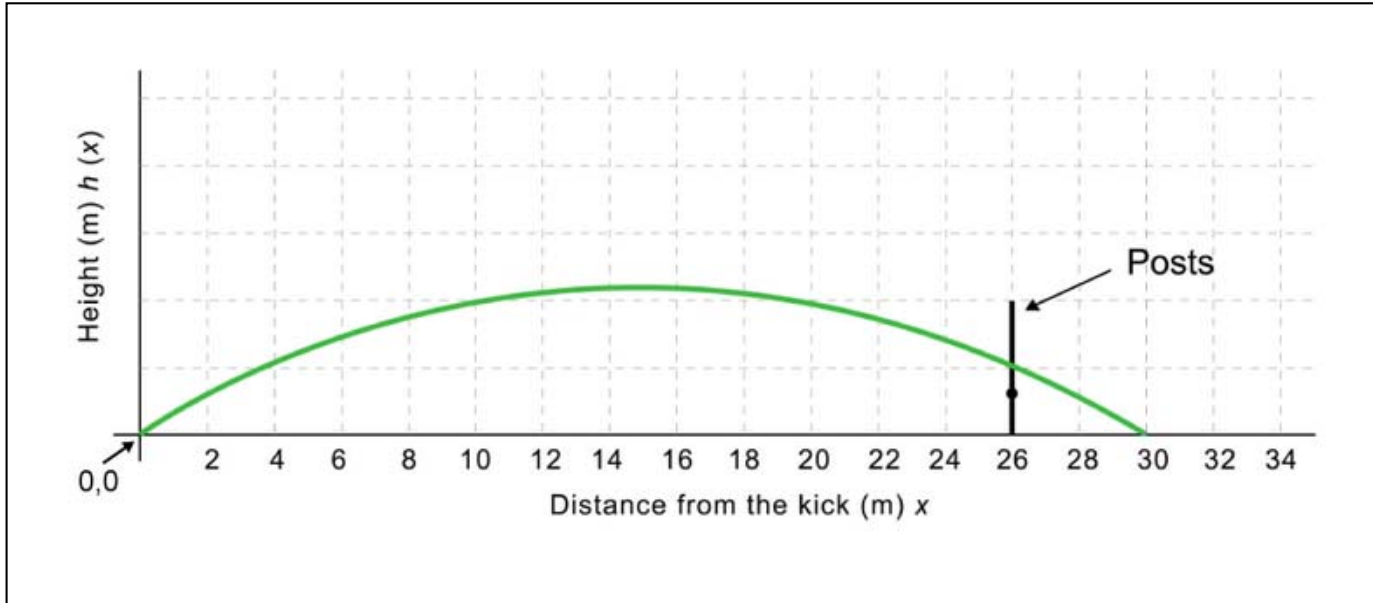|	c	Mark holistically using a best fit approach		
		Strand	**Holistic markscheme**	**Mark band**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Nothing from below	**0**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	Attempt at patterns or rules	**1 - 2**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	 One **or** more patterns in words Attempt to test one pattern	**3 - 5**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	 One **or** more patterns in words, evidence of working to find a rule, any general rule Evidence of testing the rule	**6 - 9**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	 Two **or** more patterns **or** rules found, a general rule found that **links to 8e** Attempt to prove **or** verify **or** justify a rule **linking to 8e**	**10 - 12**
		Discover patterns DIS: Describe patterns DES: Prove, verify, justify PVJ:	 Patterns **or** rules found Prove the rule from 8e **or** verify and justify the rule from 8e	**13 - 14**

(Mark band total: **14**)

Exemplification

Candidates may look at **their** developing table and find simple and more complex patterns.

Example

The denominators go 1, 2, 4, 8, 16, 32, …

The denominators are 2^n

The numerators always add to give the denominator e.g. 5 + 11 = 16

The coefficients always add to give 1

The coefficient of **a** is always smaller than that of **b**

Recognizes geometric sequence with $r = -\dfrac{1}{2}$ r

The coefficient of **a** is always nearly half that of **b**

The numerators go 1, 3, 5, 11, 21, 43, 87, …. starting at term 3

The rule for the numerator of **b** is double the numerator of **a** and then add **or** subtract 1

The numerators always add to the denominator

The numerator of **b** is the numerator of a plus twice the previous numerator of **a**

Recognizes it is a summation

$$1 - \frac{1}{2} + \frac{1}{4} - \dots$$

Example for minimum justification for $\dfrac{1}{3}a + \dfrac{2}{3}b$ the further term coefficients tends to be that the

coefficient **b** is double the coefficient of **a** **or** the limit of the sequence is $0.33333...a + 0.66666...b$

(for 9 marks)

Sums geometric sequence for coefficient of **b** $= \dfrac{2}{3}$, deduces coefficient of **a** must be $\dfrac{1}{3}$.

Concludes proof

Appendices
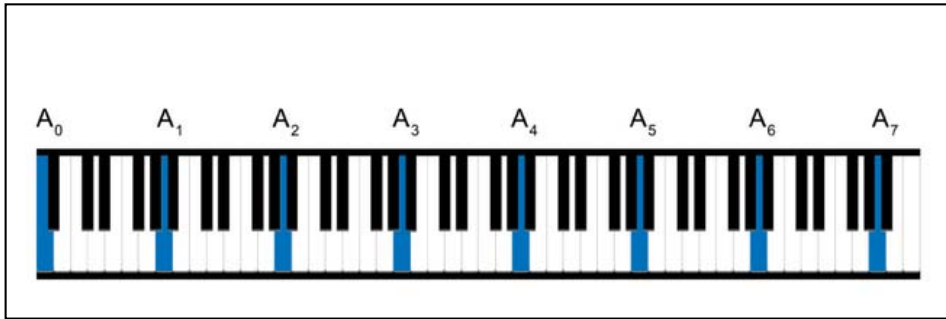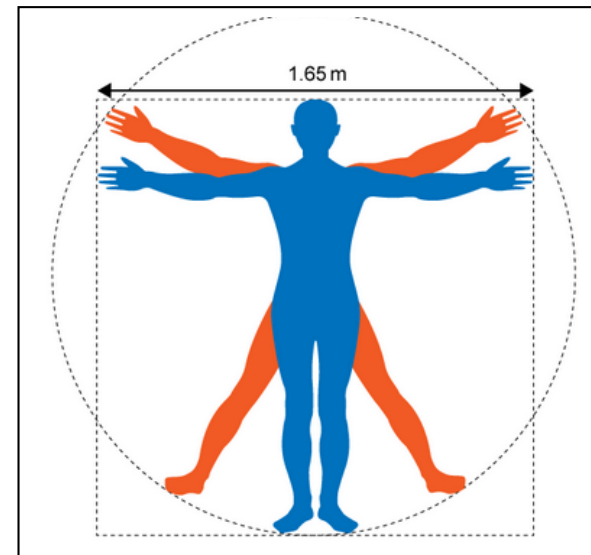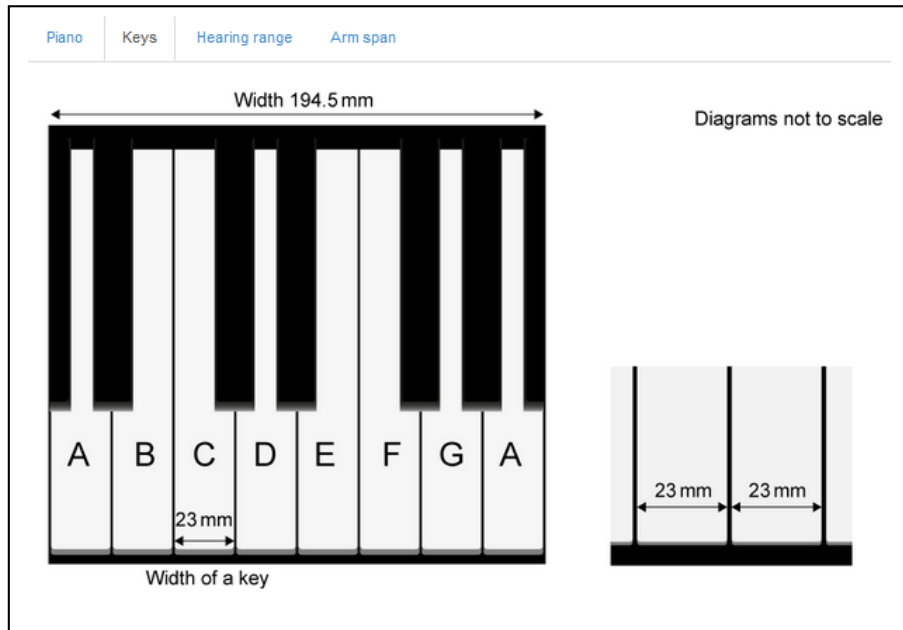
Question 4

Question 5

Question 6

Humans can only hear
between 20 and 20 000 Hertz.

Piano | Keys | Hearing range | Arm span

Grotrian-Steinweg Friedrich
Width A: 148 cm
Depth B: 52 cm
Height C: 111 cm

Piano | Keys | Hearing range | Arm span

Width 194.5 mm

Diagrams not to scale

A B C D E F G A

23 mm

Width of a key

23 mm   23 mm

1.65 m

Question 7

Tab 1   Tab 2

h

d x

s

73°   53°

d x

$d = 1.19$m

Diagram not to scale

Tab 1   Tab 2

Diagram not to scale

Temple   6 m

W_9

H

W_2

h

$W_1 = 55.3$ m